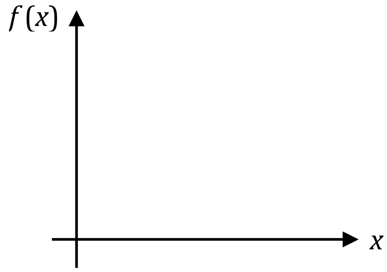
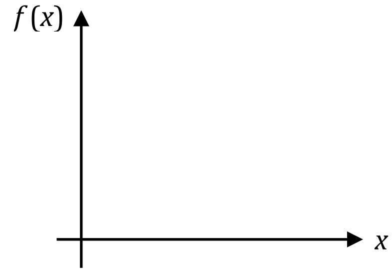
The Derivative and Tangent Line Problem

Analytically: The definition of a derivative of f at x is the difference quotient (slope generator):

$$f'(x) = m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Graphically: The derivative of a function at a point p is the slope of a tangent line to the graph of f at p.





Numerically: The derivative at a point is the *limit* of slopes of the secant lines or the *limit* of the difference quotient.

The derivative at a point $x = a$ is found by:

$$f'(a) = m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or by} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.
For all a for which the limit exists, $f'(x)$ is a function of a.

Using Limit Notation:

Let's define some specific notation for derivatives. When the function is defined as $f\ (x)$, the derivative will be written as $f'(x)$ or possibly f' (pronounced "f prime of x" or simply "f prime"). When a function is presented in "y =" form, the derivative will be given as y' or $\frac{dy}{dx}$ (read as "y prime" or "dee-y dee-x"). For now, let's think of the fraction form as a single entity representing the derivative of y.

Notation for derivatives:	$f'(x)$	y'	$\frac{dy}{dx}$	$\frac{df}{dx}$	$D_x[y]$

EX #1: Use the definition of the derivative to find the slope of the graph of $f(x) = 3x - 2$.
Find $f'(-3)$. Is the derivative a function?

EX #2: Find the slope of the tangent lines to the graph of $f(x) = x^2 - 5$ at the points $(-2, -1)$ and $(1, -4)$.

EX #3: For the function, $f(x) = x^3 - 2x$, find its derivative and evaluate the derivative at $x = 2, 0,$ and -1.

EX #4: Find $f'(x)$ for $f(x) = -\sqrt{x}$. Then find the slopes of the graph at the point $(4, -2)$. What is the behavior of the graph at $(0, 0)$?